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The state context property formalism: from concept theory to the semantics of music

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Abstract Quantum cognition is an emergent area of research that applies quantum structures in cognitive situations where traditional approaches are problematic. Independently from quantum cognition, quantum approaches for the semantics of music interpretation and improvisation have been proposed in the context of quantum computational logic and standard quantum mechanics, respectively. These frameworks overcome the non-contextual and compositional approaches to the semantics of music. In this paper, we analyze the quantum approach to the semantics of music from the perspective of quantum cognition. We first introduce an operational framework, inspired by the Brussels–Geneva approach to the foundations of quantum theory, named state context property (SCoP) formalism, that has been applied in quantum cognition to model concepts. Next, we show that the quantum approaches to the semantics of music can be operationalized in SCoP. In particular, we apply the SCoP definitions of 'orthogonal property' and 'experiment-context' to operationalize the notions of 'vague meaning' and 'interpretation', and to characterize the structural differences between the semantics of music interpretation and improvisation.

Keywords Quantum cognition · Musical semantics · Theory of concepts · SCoP · Non-compositionality

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1 Introduction

Over the last decade there has been increasing evidence for the presence of quantum structures in behavioral and cognitive processes. For example, experiments in decision making show that, in situations where people's choices are incompatible with approaches rooted in classical set-theory or classical probability, quantum approaches provide suitable models (Aerts 2002, 2009a; Aerts and Sozzo 2011; Aerts et al. 2012; Wang and Busemeyer 2013). The success of the application of quantum structures to cognition has lately been extended to a number of other fields including information retrieval (Aerts et al. 2013), natural language processing (Oehrle 2003), the dynamics of political systems (Khrennikova et al. 2014), and perception (Atmanspacher et al. 2004). This novel and interdisciplinary research program has been named quantum cognition (Bruza et al. 2013).

It is important to mention that quantum cognition does not establish a connection with the possible applications of quantum physics to explain cognition.[1] Instead it applies the quantum formalism as a language to describe situations where classical set-based approaches are problematical or inadequate.

[1] For example, quantum cognition is not based upon the assumption of quantum processes in the brain.

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One of the pioneering research areas in quantum cognition is the representation of concepts and their combinations (Aerts 2009b). In concept-combination research, one typically finds that traditional approaches (classical and fuzzy set theory, and Kolmogorovian probability) cannot supply satisfactory models for the experimentally observed patterns. Indeed, all traditional approaches to concept theory (mainly, 'prototype theory' (Rosch 1973; Rosch et al. 1976; Rosch and Lloyd 1978), 'exemplar theory' (Nosofsky 1988, 1987) and 'theory theory' (Machery 2009; Lamberts and Shanks 1997), and to concept representation ('extensional' membership-based Zadeh 1982 and 'intensional' attribute-based Hampton 1988, 1997) have structural difficulties to handle the experimental data. The reason for this failure is that the graded nature of these data strongly violates the composition of set theoretical structures (Smith and Osherson 1982; Zadeh 1982, 1965).

The quantum cognition approach differentiates from classical approaches by proposing a concept as an 'entity in a specific state changing under the influence of a context', rather than a 'container of instantiations'. This structural departure from the classical view is at the core of the success of the quantum approach, and has been operationalized in an abstract language called the state context property (SCoP) formalism. The SCoP formalism represents a concept by a set of states Σ, a set of relevant contexts $\mathcal{M}$, a set of properties $\mathcal{L}$, a state-transition function μ, and a property evaluation function ν. This approach has been applied to a number of non-classical effects including contextuality (Aerts and Gabora 2005a, b), the overextension and underextension effects in concept combination (Aerts 2009a), non-distinguishability of abstract concepts (Aerts et al. 2014), analytic versus associative thought (Veloz et al. 2011), and has been proposed as a modeling framework for information retrieval (Aerts et al. 2014; Hahn and Frank 2013).

Parallel to the development of quantum-cognition, Dalla Chiara and collaborators developed a quantum approach to model the semantics of musical interpretation (Dalla et al. 2008, 2012, 2014; Chiara et al. 2015). This approach model the interpretation of a music score using quantum computational logic (Dalla et al. 2003). Interestingly, superposition and entanglement are used as semantic resources to represent vague and non-compositional meanings usually found in music. The approach developed by Dalla Chiara and colleagues not only provides a sensible approach to represent the semantics of music, it also brings a formal account to notions that have no formal counterpart in other approaches to the semantics of music such as 'tonal ambiguity' and 'theme and variations'. Additionally, Parson, independently proposed a similar quantum-inspired approach to model the semantics of music improvisation and real-time music analysis (Parson 2012).

These models for the semantics of music strongly resemble the models developed in quantum cognition. Namely, both applications are developed to handle the imprecise and contextual notions of meaning. An interesting question is hence: can these two domains be compared from an structural point of view? Moreover, in case this comparison is possible, an even more interesting question is: how these two areas of research can be cross-fertilized?

The identification of common structural properties in seemingly different phenomena is an important, but often overlooked, scientific task. Indeed, different research areas usually develop very similar representational language for a their respective objects of study. In many cases, this is an indication that their objects of study, although seem completely different, share deep structural principles. Understanding the connections between these representational languages usually lead to synergetic advances and novel perspectives in the involved research communities.

In this work, we extend the use of SCoP from the modeling of concepts to the modeling of musical entities. In particular, we demonstrate that the approach developed by Dalla Chiara et al. can be framed using SCoP entities, and that the SCOP notions of 'orthogonal properties' and 'experiment-context' can be applied to describe musical interpretation and improvisation. In Sect. 2, we outline the classical study of musical semantics, and present the basics of Dalla Chiara's et al. quantum approach. In Sect. 3, we elaborate on the quantum inspired notion of semantic entity, introduce the SCoP formalism, and explain how SCoP can be applied to represent semantic entities in different situations. In Sect. 4, we frame the approach developed by Dalla Chiara et al. in the language of SCoP, show how SCoP enhances the representation of musical entities, and propose an abstract dynamical model that reveals the similarities and differences between music interpretation and music improvisation.

2 Musical semantics: from traditional to quantum approaches

Traditional semantic theories are rooted in Frege's principle of compositionality. This principle states that the meaning of a complex expression is determined by the meaning of its constituent expressions, and the rules to combine them (Grandy 1990). In order to be applied, this principle requires the formal specification of (1) constituent expression, which can be either logical or natural language, music, images, or any other expression (2) meaning of a constituent expression, and (3) combination rules. The principle of compositionality has an interesting advantage. It builds meaning of complex expressions from purely syntactical structures. These structures are defined a priori and are static in the theory. Thus, the quality of a compositional theory of meaning will depend only on how 'essential' the syntactical elements chosen by each particular theory are. The most convincing approach building compositional theories of meaning is found in the domain of logic, constituent expressions correspond to atomic propositions, possible meanings correspond to truth values, and combination rules are defined in terms of logical connectives. From here, model-theoretical structures can be used to build compositional theories of meaning in different fields such as natural language (Kamp and Reyle 1993), visual perception (Jewitt and Oyama 2001), and music (Zbikowski 2005).

For the particular case of a semantic theory of music, the constituent expressions are described in terms of the musical symbols used in musical scores. This includes notes and silences with different durations, accents, and other indicators, such as dynamic prescriptions, and so on (Loy 2011). The meaning of a musical composition is ruled by how these symbols are combined. These rules correspond to the longstanding effort of musical semantics (Barbar et al. 1993; Turnbull et al. 2008; Snyder 2000). Despite the success of compositional semantic theories to explain various elements of musical semantics, unfortunately it does not qualify as a formal semantic theory that can be scientifically studied or fully automatized within current representational approaches. We consider that this drawback occurs for at least three reasons. The first reason is that the most formal approaches in the semantics of music apply to western music, principally from the classical period on. Hence, we state that the semantics of music is *contextual* in its historical and geographical scene. The second reason is the absence of holistic meanings. Musical semantic models are built from notions based on either sound events or concatenations of sound events that are perceived as pleasant or somehow musical. However, there are meanings in music that are not only defined by the way in which notes are concatenated in the musical composition, but also by more complex resemblances between parts of the musical composition (e.g., 'theme and variations' Dalla et al. 2012). Therefore, we state that music possesses *non-compositional meanings*. The third reason is that some musical compositions allude to non-musical concepts such as 'hero', 'party', 'death', etc., which do not have a clear musical semantics counterpart. Hence, the semantics of music cannot be reduced to meanings such as pleasant and not pleasant, but to a much larger spectrum of sensations, possibly the entire emotional setup of humans. We could say hence that music possesses *extra-musical meanings*.

Interestingly, contextuality and non-compositionality are fundamental features of quantum systems. Quantum systems are transformed in the context of a measurement, and joint quantum systems exhibit wholeness properties that cannot be explained in terms of classical physics (Isham 2001). An abstract framework to represent these contextual and non-compositional behaviors of quantum systems is known as quantum computational logic. In quantum computational logics, meanings of sentences are represented as quregisters (systems of qubits), while logical connectives are interpreted as quantum logical gates.

Since quantum computational logics allow us to represent ambiguous, holistic and contextual meanings, these logics have been proposed to represent the semantics of music (Dalla et al. 2008, 2012).

For example, the phenomenon of tonal ambiguity can be sensibly represented in quantum computational logic. A musical passage is tonally ambiguous if its tonality is not uniquely identified. Tonally ambiguous passages are usually created by the process of modulation, a central feature in musical composition. The quantum approach represents tonalities with pure quantum states, and a tonal ambiguous passage is represented by a superposition of pure states. This superposed state embodies a probabilistic representation of the possible tonalities in the passage. Notably, quantum computational logic allow for the creation of ambiguous tonality states, from unambiguous (pure) tonality states, by means of specific quantum logical gates (Dalla et al. 2008).

A formal analysis of the concept of interpretation of a musical score has been proposed in the context of quantum computational logic. Namely, an interpretation $\mathcal{I}$ of a score $\mathbf{S}$ is defined by

$$\mathcal{I} = (Phr, Temp, Real, Wor), \tag{1}$$

where Phr is a partition of the music score into parts (or pieces) P_i, $i = 1, 2, \ldots$, named score-covering, $Temp$ is a function that assigns to any score-column of $\mathbf{S}$ a time interval, that is precise up to a certain performance limit ϵ. The map $Real$ is defined by

$$\begin{aligned} Real\colon\ & Phr \to MSpace \\ & P \to Real(P). \end{aligned} \tag{2}$$

Therefore, $Real$ assigns a musical meaning $x \in MSpace$ to any part $P \in Phr$, and Wor

$$\begin{aligned} Wor\colon\ & Phr \to WSpace \\ & P \to Wor(P), \end{aligned} \tag{3}$$

is a map that assigns an extra-musical meaning $y \in WSpace$ to any phrase $P \in Phr$.

This theory involves two spaces: the first space $MSpace$ contains the set of musical thoughts (or musical ideas). $MSpace$ embodies the musical meanings that can be associated to score-phrases in Phr. The second space $WSpace$ contains extra-musical meanings that can be evoked by the music, but that do not have a concrete counterpart in terms of musical ideas. These meanings become of crucial importance in compositions where the score includes a conceptual background, a theatrical scene, or a text such as in religious music, operas, etc. For a discussion of the structural properties of $MSpace$ and $WSpace$ we refer to Chiara et al. (2010). The set of all possible interpretations of a musical score is given by **K**. Therefore, a composition is identified with the pair (**S**, **K**).

Since in the quantum approach states can be superposed, we have that the musical meaning of a phrase $P \in Phr$ in $\mathcal{I}$ is usually given by

$$|x\rangle = \sum_{i=1}^{n} \alpha_i |x_i\rangle; \quad x_i \in MSpace \text{ for } i = 1, \ldots, n, \tag{4}$$

where $\sum_{i=1}^{n} |\alpha_i|^2 = 1$. For each $i = 1, \ldots, n$, α_i is a complex number representing the weight of the musical meaning $|x_i\rangle$ of the musical interpretation. For example, a tonally ambiguous phrase is a superposition of two (or more) musical ideas in $MSpace$ generally representing the tonalities before and after the resolution of the ambiguity (Chiara et al. 2015).

Another example of superposition is found in the notion of theme and variations. Theme and variations is a form of music that consists of a musical piece P_0, named the theme, and alterations of this piece in different ways corresponding to other pieces P_i, $i = 1, \ldots, n$. The pieces P_i are named variations of P_0 and are semantically related to the theme. Variations of P_0 can change the rhythm and the harmony, can add or eliminate notes, can change the texture or the instrumentation, or incorporate multiple changes to the P_0 at once. However, variations always resemble the musical meaning of the theme. We denote by $|Theme_i\rangle$ the musical idea that represents the meaning of P_i (with $0 \leq i \leq n$). At the same time, the vague musical idea $|Theme\rangle$ that underlies both the basic theme and all its variations can be represented as a superposition of all themes $|Theme_i\rangle$:

$$\begin{aligned} &|Theme\rangle = \sum_{i=0}^{n} \alpha_i |Theme_i\rangle; \\ &|Theme_i\rangle \in MSpace \quad \text{for } i = 0, \ldots, n, \quad \text{and} \\ &\sum_{i=0}^{n} |\alpha_i|^2 = 1. \end{aligned} \tag{5}$$

For an example of the representation of extra-musical meanings, consider the case of leitmotives in opera compositions. A leitmotiv is a generally short theme, that may be semantically associated to a character, to a place, to a situation, etc. In the structure of an opera, each leitmotiv appears in different musical and theatrical forms. Therefore, we can represent the leitmotiv as a superposition $|Leitmotiv\rangle$ of musical and extra-musical meanings that belong to the tensor-product space $MSpace \otimes WSpace$. In some interesting cases, $|Leitmotiv\rangle$ might correspond to what in quantum theory is known as an entangled state (Chiara et al. 2010). These states exhibit non-classical correlations that represent their holistic structure. When $|Leitmotiv\rangle$ is entangled, one can say that musical ideas co-determine some corresponding extra-musical meanings, and vice versa.

A simplified version of this quantum approach to the semantics of music appeared independently in the field of musical improvisation (Parson 2012). In this approach, a musical improvisation is a sequence of superposed event states, and performance consist of the collapse of these superposed states, thereby cascading and emitting music, and creating new states to be collapsed later. Conventional musical notation takes this approach to its limit by setting the superposition weights to 1 or 0. The quantum approach to musical improvisation extends the traditional approach by allowing for superposed states to have intermediate probabilities. This suggests the possibility to improvise following a bi-directional and hierarchical network of musical events (Parson 2012). At the most concrete level of this hierarchy we encounter sounds, at the next level we have musical meanings representing harmonic and rhythmic properties of sound sequences similarly to the elements of $MSpace$. Next, there is a hierarchy of extra-musical meanings that allude to concepts at different abstraction levels, similar to $WSpace$. This hierarchical network is the substrate where the semantics of musical improvisation is formed. The collapse from abstract to concrete states, and the regeneration of abstract elements of the superposed state entails the dynamics of musical improvisation. This approach has been shown to be relevant for the analysis of real-time music performance and for automated music generation. For details we refer to Parson (2012).

3 The quantum semantic entity

Along with the development of a quantum approach to the semantics of music, the notion of semantic entity has been introduced in Aerts et al. (2014) to capture the most abstract, but still formally tractable, form of cognition. A semantic entity can be an idea, a decision, a piece of text, or any perceivable situation that underlies a cognitive phenomenon. In order to put forward a formal notion of what a semantic entity is, certain structural aspects that are common to the expressions any cognitive phenomena must be clarified. First, for every cognitive phenomenon we assume that it is possible to refer to it as an element having a certain structure. This means to assume the existence of semantic entities. Second, we assume that this entity can exist in a number of different ways. These ways however, do not necessarily correspond to the concrete manifestation of the entity in reality. Particularly, a semantic entity can exist in a *vague* state. Third, the manifestation of the entity in reality is by a non-vague (concrete) state. Concrete states are what we are able to observe about the cognitive phenomenon. However, these states do not completely characterize the cognitive phenomenon, since these concrete states are the result of the interaction between the most primitive vague states and the circumstances by which the entity is evoked/expressed.

We will give an example that clarifies the generality of our assumptions. Suppose a person wants to express an idea that he/she has in mind using words (either speaking or writing). We call this idea a semantic entity. Hence, the first structural principle is met. Second, notice that an idea prior to being expressed has a clear meaning to the person, so we can say the person knows what is going to be said. However, the exact wording of the idea is not clear. Indeed, the idea can be expressed using different words and grammatical structures, as well as different attitudes and sentiments, etc. Therefore, the idea can be identified with a vague state. Hence, the second structural principle is met. Third, the manifestation of the idea, i.e., the expression of the idea, will be aligned with the situation in which the person is encountered. The description of the person's situation involves a myriad of parameters including the language in use, the physical and emotional circumstances of the person, the time available to express the idea, etc. Despite the complexity of the characterization of such circumstances, the third structural principle is also met. Thus, we conclude that semantic entities meet these three structural principles.

Note that the vague states of an idea are structurally different to their concrete manifestations because the vague idea embodies the potentiality of multiple expressions, and the concrete manifestation corresponds to only one of those expressions. Hence, we can conclude that the vague states of a semantic entity cannot be directly observed. Instead, we can observe their concrete manifestations only. Moreover, we know that these concrete manifestations are the result of the interaction between the semantic entity and circumstances that brought the semantic entity into a concrete form in reality. Accordingly, the question is how we manage to reconstruct semantic entities by only accessing to states that correspond to their contextual manifestations?

Remarkably, the reconstruction of entities from their contextual manifestations is exactly what occurs in quantum particle laboratories such as CERN (Irvine and Martin 1984). Namely, the quantum particles are observed by the trace they left in particle detectors, after interacting with an experimental context involving usually other particles and fields. In order to reconstruct the quantum particles' states, physicists use a mathematical formulation called 'inverse problem' to reconstruct the quantum entities from the traces they left at the particle detectors (Chadan and Sabatier 1977). This is the current methodology to observe the most fundamental particles known the physical realm.

It has been proposed in Aerts et al. (2014) that the inverse problem approach can be applied to build semantic entities. Traces of semantic entities can exist in the form of text, sounds, images, purchases, or any other perceivable manifestation of a semantic entity resulting from its interaction with a context. The context is defined according to the field of application. The specification of how to detect and model the traces of semantic entities is beyond the scope of this article. However, we present a mathematical formalism, developed by the Brussels' school of quantum cognition, that distills the structural properties of semantic entities, and hence can be applied to operationalize the inverse problem for semantic entities.

3.1 Semantic entities in the SCoP formalism

The SCoP formalism is an operational approach rooted in the foundations of quantum theory, and is part of a longstanding effort to develop an operational approach to quantum theory known as the Geneva–Brussels approach (Piron 1976). In SCoP, a physical system is determined by the mathematical structure of its set of states, set of properties, and the possible (measurement) contexts which can be applied to this entity. In addition, if a suitable set of quantum axioms is satisfied, one recovers via the Piron–Solèr representation theorem the standard quantum mechanics (Piron 1976). The SCoP formalism is an operationalization of how the interaction between context and the state of an entity plays a fundamental role in its evolution. For this reason, SCoP has been used to describe not only physical entities, but also semantic entities such as concepts (Aerts 2002; Aerts and Gabora 2005a, b; Gabora and Aerts 2009), and webpages (Hahn and Frank 2013).

Formally, a SCoP entity consists of three sets Σ, $\mathcal{M}$, and $\mathcal{L}$, named the set of states, the set of contexts, and the set of properties, respectively, and two additional functions μ and ν. Through this section we denote states by letters p, q, contexts by e, f, and properties by a, b. The function μ is a transition function that describes how likely it is that state $p \in \Sigma$ under the influence of context $e \in \mathcal{M}$ changes to state $q \in \Sigma$. Mathematically, this means that μ is a function from the set $\Sigma \times \mathcal{M} \times \Sigma$ to the interval [0, 1], where $\mu(q, e, p)$ is the probability that state p under the influence of context e changes to state q. We write

$$\mu\colon \Sigma \times \mathcal{M} \times \Sigma \to [0, 1] \qquad (q, e, p) \mapsto \mu(q, e, p), \tag{6}$$

$$\sum_{q \in \Sigma} \mu(q, e, p) = 1 \quad \text{for all } p \in \Sigma,\ e \in \mathcal{M}, \tag{7}$$

ν is a binary function that describes whether or not a property is held by a state.[2] This means that ν is a function from the set $\Sigma \times \mathcal{L}$ to the set $\{0, 1\}$, where $\nu(p, a) = 1$ if property a is held by the state p, and $\nu(p, a) = 0$ if not. We write

$$\begin{aligned}\nu \colon \Sigma \times \mathcal{L} &\to \{0, 1\}\\ (p, a) &\mapsto \nu(p, a).\end{aligned} \tag{8}$$

Thus, a SCoP entity S is defined by the five-tuple $S = (\Sigma, \mathcal{M}, \mathcal{L}, \mu, \nu)$. We introduce a *unitary context* $\mathbf{1} \in \mathcal{M}$ that does not induce transition for any state, and a *potential state* $\hat{p} \in \Sigma$ which represents the state of an entity prior to any contextual interaction. Thus

$$\begin{aligned}&\mu(p, \mathbf{1}, p) = 1 \quad \text{for all } p \in \Sigma,\\ &\mu(\hat{p}, e, \hat{p}) = 0 \quad \text{for all } e \in \mathcal{M} - \{\mathbf{1}\}.\end{aligned} \tag{9}$$

The unitary context and the potential state correspond to the identity operator and to a maximally mixed state in the standard quantum formalism. Other relevant notions of SCoP involving contexts and states are the notions of 'experiment-context' and 'eigenstate'. For a given context $e \in \mathcal{M}$ we say $p \in \Sigma$ is an eigenstate of e if and only if $\mu(p, e, p) = 1$. A context such that all the state transitions are eigenstates of itself, is called an experiment-context, or 'experiment'. Formally, a context e is called an experiment if and only if for all $p \in \Sigma$:

$$\text{If } \mu(q, e, p) > 0, \text{ then } q \text{ is an eigenstate of } e. \tag{10}$$

An experiment embodies the notion of a projection operator in the standard quantum formalism.

Note that every state $p \in \Sigma$ can be characterized by the set of properties that holds. For each $p \in \Sigma$ we define

$$\mathcal{L}_p = \{a \in \mathcal{L} \text{ s.t. } \nu(p, a) = 1\}. \tag{11}$$

Analogously, we can characterize properties as sets of states. For each $a \in \mathcal{L}$ we define

$$\Sigma_a = \{p \in \Sigma \text{ s.t. } \nu(p, a) = 1\}. \tag{12}$$

Therefore, we can identify states as collections of properties and vice versa. This duality allow for the definition of orthogonal property. A property a is orthogonal to a property b with respect to an experiment e, denoted by $a \perp_e b$ if and only if for all p eigenstate of e

$$\begin{aligned}&\nu(p, a) = 1 \text{ implies } \nu(p, b) = 0, \quad \text{and}\\ &\nu(p, a) = 0 \text{ implies } \nu(p, b) = 1.\end{aligned} \tag{13}$$

[2] The function ν can be extended to graded evaluations. For simplicity, we present ν in its binary form.

Next, two properties a and b are orthogonal, denoted by $a \perp b$ if $a \perp_e b$ for all the experiments e in $\mathcal{M}$. Note that after an experiment, the resulting state cannot possess two orthogonal properties measured by the experiment. Indeed, orthogonal properties embody the notion of quantum numbers in standard quantum mechanics. Orthogonal properties cannot be obtained in the outcome of an experiment, but still can be the result of a contextual transition. Therefore, states possessing orthogonal properties represent vague (non-concrete) meanings. In the next section, we will focus on how the notion of vague meanings in SCoP is relevant to the quantum-inspired musical approaches presented in sect. 2.

4 Quantum musical semantics and SCoP

The formalism of SCoP can be applied to the semantics of music model elaborated in Dalla et al. (2008, 2012). This application is inspired by the structural resemblance between the notion of interpretations, defined in Sect. 2, and the notion of SCoP semantic entity elaborated in Sect. 3. Namely, both interpretations and semantic entities respond to the same structural principles: (1) contextuality, (2) vague meanings, (3) and non-compositionality. Therefore, we propose that the set of interpretations $\mathbf{K}$ is a semantic entity.

Recalling that an interpretation $\mathcal{I} \in \mathbf{K}$ encapsulates the meaning of the score $\mathbf{S}$, we can hence assume that the meanings of the possible interpretations in $\mathbf{K}$ can be represented by states $p_{\mathcal{I}} \in \Sigma_{\mathbf{K}}$, and that the state representing the musical meaning prior interpretation is given by the potential state $\hat{p}$, that depends on the score $\mathbf{S}$. A particular interpretation depends on the interpretation context. The interpretation context involves the available resources and motivations that derived to conceive the interpretation. This includes available artists for performing, the place where the performance will occur, among other aspects such as the intention to innovate, or instead to try to express a piece in its traditional form. We denote this set of contexts my $\mathcal{M}_{\mathcal{I}}$.

Similarly, the concrete manifestations of an interpretation, i.e., the performances, can be identified with a state $p_C \in \Sigma_C$. Interestingly, p_C can be identified as a collapsed state from the interpretation state $p_{\mathcal{I}}$. The contextual factors related to this collapse include the particular configuration of the stage, personal factors of the performers, etc. All these factors define the context set of a performance $\mathcal{M}_C$. Therefore, we define the probability to obtain a particular performance p_C in a context e by $\mu(p_C, e, p_{\mathcal{I}})$.

The specific structure of an interpretation developed in Sect. 2 can be represented using SCoP. In particular, the score-covering $Phr(\mathbf{S}) = (P_1, P_2, \ldots, P_n)$, as well as musical and extra-musical meanings determined by $Real(P_i)$ and $Wor(P_i)$ are encapsulated in the interpretation state $p_{\mathcal{I}}$. Namely, $p_{\mathcal{I}}$ corresponds to a sequence of states

$(p_{\mathcal{I},1}, p_{\mathcal{I},2}, \ldots, p_{\mathcal{I},n})$,

representing the interpretation state of the part P_i. Moreover, the tempo specified by the image $[0,\infty]$ of $Temp$, and the musical and extra-musical meanings specified by the images $MSpace$ and $WSpace$ of the functions $Real$ and Wor, respectively, correspond to properties in $\mathcal{L}$. Thus, these properties can be identified in the interpretation states by the function ν. In particular, the function ν restricted to the domains $\Sigma \times [0,\infty]$, $\Sigma \times MSpace$, and $\Sigma \times WSpace$ recover the functions $Temp$, $Real$, and Wor, respectively.

Since the performance state p_C is a collapsed state of $p_{\mathcal{I},i}$, then

$$p_C = (p_{C,1}, \ldots, p_{C,n}).$$

The concrete state p_C does not have unambiguous meanings. For example, $p_{\mathcal{I},i}$ is identified with a duration that has a precision limit given by ϵ. However, the collapsed state $p_{C,i}$ has a concrete duration that is exact, i.e., a value of $[0,\infty)$. This means that the duration of P_i in $p_{\mathcal{I},i}$ is vague (up to a certain limit), but in the concrete state this duration is precise. Therefore, for a performance state the elements in $[0,\infty)$ are orthogonal properties. However, this is not necessarily the case for the interpretation states.

Usually, interpretation states contain ambiguous meanings. These meanings are represented by superposition of unambiguous meanings. The degree of superposition of ambiguous meanings lies between the complete state of potentiality, determined by the state $\hat{p}$, and the performance states (where no meaning is unambiguous). Therefore, for all $p \in \Sigma_C$, we have that $a, b \in \mathcal{L}_p$ implies $a \perp b$.

From here, the SCoP formalism provides a connection between the approaches to the semantics of musical interpretation and to the semantics of music improvisation presented in Parson (2012). In particular, the hierarchy of musical semantic states in Parson (2012) is a possible specification of the structure of musical thoughts and vague worlds described by $Real$ and Wor, respectively, in Dalla et al. (2012). In SCoP, we can build a hierarchy of states from the number of orthogonal properties they hold. Namely, the more orthogonal properties are held by a state, the more abstract the state is. Since states with orthogonal properties are superposed states, we have that at the most abstract level we encounter the potential states $\hat{p}$, that holds all the orthogonal (and non-orthogonal) properties, and at the most concrete level we have the performance states in Σ_C.

We can also identify some structural differences in the performance process of music interpretation and music improvisation. Note that music performance is in both processes is modeled as a sequence of transitions from abstract (superposed) to concrete performance states (Dalla et al. 2012; Parson 2012). Using the SCoP language, we are able to clarify two differences between these two kinds of musical performance: The first difference is that the performance state obtained from an interpretation state $p_{\mathcal{I}}$ is associated to the score **S**, while in musical improvisation the improvisation state is not associated to any score. Indeed, improvisation is normally developed over a melody or rhythm. Therefore, the improvisation state $\hat{p}_V$ is similar to theme state in Eq. (5), except that in the case of improvisation this sum is infinite. The second difference is that the contextual transitions in the interpretation of a score entail the collapse of pieces P_i into states $p_{\mathcal{I},i}$, while for improvisation there is a dynamic process of collapse and creation of states that is repeated continuously.

To be more specific, consider a score **S** with a score covering $(P_1, \ldots, P_n)$. The state $p_{\mathcal{I},1}$ of P_1 collapses to $p_{C,1}$ by interacting with the performance context. Let's call this context e. This transition has probability equals to $\mu(p_{C,1}, e, p_{\mathcal{I},1})$. The same transition process continues until the piece is finished by the last transition from $p_{\mathcal{I},n}$ to $p_{C,n}$ with probability $\mu(p_{C,n}, e, p_{\mathcal{I},n})$. The improvisation occurs in a similar way. However, it begins from a potential state $\hat{p}_{V,0}$ that is not connected to any score (but to the extra-musical meaning of the music the improvisation is played over), and it occurs in a collapse–creation fashion. For any improvisation round $i = 1, \ldots, n$, the improvisation state $\hat{p}_{V,i}$ collapses to a performance state $p_{V,i}$, and a new potential state $\hat{p}_{V,i+1}$ is attached to the performance state $(p_{V,0}, \ldots, p_{V,i})$ forming a new state $(p_{V,0}, \ldots, p_{V,i}, \hat{p}_{V,i+1})$. This process is not bounded in time, as it can continue over and over in the same way improvised music does.

5 Conclusion

We have introduced the SCoP formalism to represent quantum features found in music semantics. The structural properties of music semantics, namely contextuality, non-compositionality and the existence of extra-musical meanings, suggest that traditional approaches based in classical logic and probability are not candidates to model the semantics of music. As an alternative, we reviewed the quantum approach to musical semantics of a score interpretation developed in Dalla et al. (2008, 2012), and a similar approach to the semantics of music improvisation (Parson 2012). Next, we introduced the SCoP formalism, which is an operational formalism rooted in the foundations of quantum theory to model general semantic entities such as concepts and decisions, and extended its application to musical entities. We showed that it is possible to frame the quantum-inspired notion of music interpretation developed in Dalla et al. (2008, 2012) in the SCoP formalism, and that this view is compatible with the approach to music improvisation developed in Parson (2012).

In particular, we applied the SCoP notions of ‘orthogonal property’ and ‘experiment’ to identify states and transitions that either correspond to unambiguous music interpretations (performances), or encapsulate vague meaning (interpretation and improvisation). These latter states are crucial for the representation of the semantics of music.

An interesting question for further exploration is how meaning is constructed by the audience of a music performance, if they only have access to the performance states? The audience of a musical performance creates a state that is structurally similar to the interpretation state, but from the collapsed states corresponding to the performance. Therefore, each person in the audience carries out an inverse problem to create a musical semantic entity from the performance. If the person ‘solves’ successfully this inverse problem, the state of his musical semantic entity embodies the vague and extra-musical meanings in the interpretation state of the music director. We plan to investigate the ‘inverse-problem aspects’ of musical understanding in future research.

Quantum cognition proposes an interesting alternative to classical approaches, specially in situations where vagueness and non-compositionality are fundamental structural features. However, it is important to consider that quantum cognition might find an obstacle due to the multiplicity of mathematical languages developed to represent quantum structures.[3] In particular, due to the interdisciplinary nature of cognitive phenomena, several researchers untrained in quantum theory are involved. Therefore, it is important to moderate the use of different quantum theoretical languages in cognition, and clarify similarities and differences when necessary. In this paper, we applied the abstract operational formalism of SCoP to establish the similarities and differences between an approach to musical semantics rooted in quantum computational logic (Dalla et al. 2012), and another approach rooted in standard quantum mechanics (Parson 2012). We believe that this type of integrative work is necessary to foster the interdisciplinary cross-fertilization between different disciplines, and to expand the span of this promising research program.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

[3] For example, the language of standard quantum mechanics is used for physical systems and teaching, quantum-logic is used in logic, quantum-information is used in computer science and some areas of physics, the C$*$-algebra formulation is used in mathematical physics, etc.

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