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(Re)interpreting $E = mc^2$

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Abstract
We propose a new interpretation of the equation $E = mc^2$ in special relativity by generalizing ideas of ontological emergence to fundamental physics. This allows us to propose that mass, as a property, can be considered to emerge from energy, using a well-known definition of weak ontological emergence. Einstein's famous equation gains in this way a clearer philosophical interpretation, one that avoids the problems of previous attempts, and is fully consistent with the kinematic properties of special relativity, while yielding fresh insights concerning the nature of mass.

Keywords Interpretation of physics · Special relativity · Emergence · Higgs mechanism

1 Introduction

Starting from the relativity principle and the Maxwell–Hertz equations (which entail the invariance of the speed of light), Einstein deduced from a *Gedankenexperiment* a reduction of the mass of a body after the release of energy in the form of radiation, which turns out to be proportional to the emitted energy. Einstein [10] concluded that:

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> If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

This was the seed to the formulation of $E = mc^2$, arguably the most famous equation of all time. The ubiquity of this formula does not come however with a clear understanding of its meaning. Indeed, after more than a century this famous equation still has no universally accepted interpretation. Einstein developed a mathematical demonstration of the relationship between mass and energy, but there is still controversy as to what kind of relationship this equation stands for. For some authors it makes no sense to ask about the kind of relationship that exists between mass and energy because only one of those properties is a real property (whereas the other is not), and therefore they cannot be related [17]. On the other hand, some authors claim that energy and mass are one and the same property, holding therefore an identity relationship [9, 31, p. 146]. Still others argue either that energy and mass should be regarded as different properties capable of interconverting [12, 28], or that they are different properties that do not have this capability of interconvertibility [3].

In this work we offer a new take on this, by making use of the notion of emergence—generally reserved for the consideration of macroscopic systems. Thus, we argue that the right way to interpret $E = mc^2$ is that mass, as a property, emerges from the more fundamental property of energy. This aligns well with the view from contemporary physics, in which mass is seen as a dynamical quantity instead of an intrinsic property of physical objects. A famous example of this is given by the Higgs mechanism in the standard model of particle physics, which we discuss below as one of our motivations, and which has already been used by Bauer [2] to argue against considering mass as a fundamental, un-grounded property. On top of that, the notion that mass is 'emergent' in some undefined sense is commonplace in the physics community, see e.g. Wilczek [34, 35].

Emergence itself is a fraught concept, so it is not a priori clear if the characterisation of mass as emerging from energy is illuminating. For this reason we choose to stick to one of the proposed meanings of 'emergence' in the literature that has withstood the test of time, originally conceived to represent the emergence of mental properties. That this notion of emergence is able to deal with such disparate cases as the mind and subatomic particles is, in our opinion, an additional argument for the soundness of the definition. By sticking to one conception of emergence we are excluding many others that might be incompatible with our proposal. But this is unavoidable given the current state of the emergence literature. To try to accommodate all possible notions of emergence would be nothing short of paralyzing.

The concept of weak emergence of Macdonald and Macdonald [21], stipulates that there is no emergence of substances but only of properties, and identifies each instance of a higher level property with a lower level property, while avoiding the confusion between properties and their instances, and turns out to be very apt to describe the relationship between mass and energy. Mass as a determinable property is independent from energy, but every determinate value of mass for a physical system is generated by a given energy content, where energy is assumed to be the 'basal' property. The fact that some physical systems can have energy while having a zero value of mass is what justifies this ontological priority. In other words, no physical system has zero energy, while it is possible to find physical systems with zero mass[1].

If our proposal is accepted, the notion of emergence could consequentially be extended to one of the most fundamental frameworks in science, the theory of special relativity. Thus, we would not only have a clearer understanding of the physical meaning behind Einstein's $E = mc^2$, but we would also have valuable information for further inquiries into the nature of emergence itself.

This work is organized as follows: in Sect. 2 we go over the interpretations of $E = mc^2$ that have been proposed so far, which can be grouped in four different classes. As we show below, the emergent interpretation can be considered a special case of one of these classes. In Sect. 3 we give a brief overview of the notion of weak emergence, which we believe holds the key for interpreting $E = mc^2$. After this groundwork we can explicitly advance our interpretation in Sect. 4, where we show how considering mass as a property that weakly emerges from energy yields a view that avoids the issues of all previously available interpretations. After this, in Sect. 5 we take a detour into more recent physics to show how the Higgs mechanism in the standard model of particle physics is also compatible with taking mass as a weakly emergent property. We give our conclusions in Sect. 6.

2 The Challenge of Interpreting $E = mc^2$

In order to begin our discussion, it is important to give a definition of the physical meaning of each term in the equation $E = mc^2$. Mass, as a property, can be associated to objects and it measures the inertia, or resistance to change the movement state, of every object. Energy, conversely, is best understood as a property of systems. To make this important difference more explicit: properties in the world can be associated either to individual objects, or to systems. Usually, when we analyze systems into their constituting components, some systemic properties are seen to be relational (as opposed to intrinsic) properties of the components. This is the case with energy.

Note that macroscopic objects can be considered as systems of interacting components depending on the level of description or, equivalently, on the relevant scale of energy under analysis. Whenever the binding energies between the components of the system (as defined by its Hamiltonian) is much larger than the energy scales

[1] Note that, within classical mechanics, it is always possible to choose a frame of reference and the zeros of potential energy in such a way that, at a given time along the evolution of a system, the net value of its total energy is equal to zero. Here we refer to a deeper fact revealed by special relativity, namely that the invariant mass of some physical system, i.e. the modulus of the energy-momentum four-vector, can be zero although its total energy cannot be zero for any non-trivial case.

of interest for the study of a certain phenomenon, the system can be considered as a simple physical object when analysing this phenomenon.

A system of particles has a certain energy, which is a relational property of the positions and velocities of every particle, whereas the energy of each particle in isolation is usually ill-defined[2]. On the contrary, each particle does singularly have a well defined mass, corresponding to its inertia. The whole system of particles can be assigned a value for its inertial mass as well, which generally speaking will be different from the sum of the component masses—more on this is said in what follows.

On top of these basic definitions, care must be taken when considering the frame dependence of equations in special relativity (from now on abbreviated as SR). As correctly pointed out by Okun [26], the meaning given to the terms E and m in $E = mc^2$ varies throughout the literature, with both quantities sometimes standing for Lorentz invariant properties (the rest energy and mass), and sometimes for frame dependent properties. The notion of a velocity- or frame-dependent mass is quite unjustified, and mostly a relic of trying to preserve the no longer valid Newtonian equations in a relativistic setting [26].

The general relation between energy and mass in SR is given by the equation

$$E^2 - \mathbf{p}^2 c^2 = m^2 c^4 \tag{1}$$

where E is the total (frame dependent) energy, $\mathbf{p}$ is the 3-momentum defined as $\mathbf{p} = \mathbf{v}\frac{E}{c^2}$, and m is the invariant rest mass. The fact that m is a Lorentz invariant follows trivially from the 4-vector structure which relates energy and momentum, analogous to the structure made by their canonical conjugates, time and space.

The famous equation is nothing else than Eq. (1) expressed in the rest frame. Thus, the natural meaning for E and m are the values at rest, and not the velocity dependent values. Nonetheless, given the reigning confusion in the literature, we emphasize that when m is written in an equation, it refers to rest mass. For future reference, notice in particular that $E = mc^2$ is not a valid equation in cases where $m = 0$, as it is impossible to go to the rest frame of a massless particle.

Even when considering these correct meaning for its terms, several incompatible proposals have been put forward as to how to interpret our preferred equation. In turn, all these proposals can be organized in four mutually exclusive interpretative approaches, of which we give a succinct overview.

In the first place, $E = mc^2$ has been claimed to show a strict equivalence between mass and energy, so that mass would be *identical* to the energy of a body at rest, and, given the interconvertibility of energy, mass and energy would be generally equivalent, that is to say, coreferential. In Eddington's words:

> It seems very probable that mass and energy are two ways of measuring what is essentially the same thing, in the same sense that the parallax and distance of a star are two ways of expressing the same property of location [9, p. 146].

[2] An individual particle can have kinetic energy, but this is only once a reference frame, or idealized observer, is set—the particle has a kinetic energy *with respect to* a certain reference frame—i.e. a relational property.

Although intriguing, the identity interpretation cannot be justified simply by the existence of the relation $E = mc^2$, in the same way as Planck's equation $E = \hbar\nu$ does not entail an equivalence between energy and frequency, nor $m = \rho V$ between mass and density. That is to say, in order for two properties to be one and the same, it is not enough to show that they are related by an equation, and even more so when they do not even seem to belong to the same ontological class: as stated before, whereas mass is ostensibly a property of physical *objects*, energy is mostly considered to be a property of physical *systems*. An identity thesis is a metaphysical thesis that cannot be sustained only by appeal to a physical equation [20].

In fact, there are a variety of good arguments against this straightforward identity interpretation. Famously, [17, 18] pushes against it. According to this second view, energy and mass cannot be related by identity because one of them is a real property of physical bodies, while the other is an illusion arising from our analytical shift from the level of parts to the level of wholes (see [13]). In a nutshell, the argument is that energy is not a Lorentz invariant quantity, i.e. it is different for different observers depending on their reference frame. Therefore, according to Lange, as long as energy depends on arbitrary choices, it cannot be considered as a real entity. Only quantities independent of reference frame choices can enjoy 'objective reality' (see also [33, p.132]):

> The Lorentz invariant quantities are exactly those which depend only on how the universe really is, uncontaminated by any contribution from us in describing the universe. [18]

Lange's interpretation of $E = mc^2$ can best be taken as a matter of ontological priority as opposed to a question of the strict non-reality of the energy [6]. In other words, energy, as one of the components of the 4-momentum, is indeed a physical quantity (just as 3-velocity is) but one that has to be derived from a Lorenz invariant, instead of being a fundamental quantity. The issue stands: a frame invariant quantity (rest mass) cannot be identified with a derived, frame dependent one (energy).

Unfortunately, this approach clashes with some well-known cases of mass being generated out of energy in SR. A clear example, a gas of relativistic particles, is discussed below. But a simpler example is given by the treatment that Einstein gave in some of the first introductions to $E = mc^2$: a completely inelastic collision between two bodies, in which the total mass after the collision is greater than the sum of the initial masses. In this case, it is difficult to deny that part of the final mass comes from energy..

Even so, Lange gives a strong argument to consider mass and energy as different properties. The question then becomes what is happening in cases such as above, when we see energy becoming mass or vice-versa. Surprisingly, there is also no consensus on this point: some authors (e.g. [12]) argue that mass and energy are different properties that can convert into one another, whereas others, most famously Bondi and Spurgin [3] argue that no such convertibility takes place. Both these views come with a set of issues.

One important antecedent to understand the no-conversion interpretation defended by Bondi and Spurgin [3] (but not only, see also [16]) is the fact that some processes of conversion between energy and mass are better understood as transformations between different kinds of energy. An example of mass transformation into kinetic energy that is better understood as potential energy transforming into kinetic energy can be found in processes of nuclear fission, where the difference in mass between the original nucleus, and the sum of masses of the final parts plus the mass of the liberated neutrons is usually regarded as the amount of mass converted into energy. This transformation can be understood as a change of nuclear potential energy carried out by strong forces into kinetic energy of the remaining nuclei and neutrons after fission, with no rest-mass converted into energy. In this view energy 'contributes' to mass, or the mass of an object 'depends upon' its energy content. More succinctly, though perhaps imprecise, one could say that energy 'has' mass. Energy and mass are, by this proposal, always conserved separately[3].

For this to be consistent, Bondi and Spurgin [3] have to define mass not as rest mass, but as the frame dependent quantity $m\gamma$—usually called the relativistic mass—going against our general warning above. This is how photons have 'mass', as they have energy, and how this 'mass' is conserved when they make up the photon gas mentioned above. But this is problematic, because mass would become an observer-dependent quantity, instead of an intrinsic property of physical objects. By the same token, as Lange argues so well, this frame dependent property cannot be considered real (or at least fundamental: it is real as a quantity derived from the rest mass).

Still, there are good intuitions behind Bondi and Spurgin [3] proposal, also when considered as a claim about the relationship between energy and rest mass. As they say: "Mass and energy are not interconvertible. They are entirely different quantities and are no more interconvertible than are mass and volume, which also happen to be related by an equation" [3]. Below, we show how our interpretation does justice to these intuitions.

If we state things in terms of the rest mass, however, the no-conversion interpretation has some difficulties dealing with universally accepted phenomena, the most important of which is pair creation and annihilation in particle physics. Indeed, it is not trivial to explain, from the point of view of taking conversion as impossible, how mass can appear or disappear from the world while at the same time requiring or generating the exact amount of energy indicated by $E = mc^2$. This is compounded with the possibility—clearly compatible with SR—that fundamental particles with non-zero rest mass exist, so that not all of the mass in the universe depends upon its energy content.

As for the second option, if there is genuine conversion between mass and energy, necessarily "a certain amount of one [mass or energy] 'disappears' and an equivalent amount of the other 'appears'" [12]. That is, one has to disappear while the other appears, literally. But this makes appeal to an unknown, and by all means bizarre, kind of process capable of making a relational property of systems (energy) disappear and transform into a completely new property of individual objects (mass),

[3] One could see this interpretation as equivalent to saying that mass is just an illusion, and that energy is all there is to matter. But inertia is a measurable, real property, different from the energy of an object, and mass is just a quantification of inertia.

and vice-versa. The conversion interpretation requires a problematic kind of physical process of which nothing is said besides its compatibility with pair annihilation.

Recently, Coffey [6] makes an argument for a a new interpretation of $E = mc^2$, by advocating for the ontological priority of 4-vectors—in particular 4-forces and 4-velocities—over energy and mass, which would both be derived or secondary properties. The proposal is sound, but has the inconvenience of assigning ontological priority to highly abstract objects, namely 4-vectors in Minkowski space-time. Our approach is to be parsimonious at the time of proposing ontological categories, and also to be wary of directly reifying mathematical structures. Still, we believe that our proposal can complement and contrast with Coffey [6] in interesting ways.

Having gone over the different conflicting views on the meaning of $E = mc^2$ and the issues involved, it is time to approach our proposal: the emergence of mass from energy. For this, we have to introduce the relevant notion of emergence.

3 Weak Emergence

The usual arguments for the non-fundamental character of mass are well covered by Wilczek [35]. The Nobel laureate claims that mass 'emerges', although the appeal to 'emergence' is meant to capture the important notion of novelty, rather than to make explicit reference to the (related) philosophical concept. Some key features are left unmentioned; for instance, what corresponds to the upper or lower level of description is not specified, nor what is the basal property (or properties) upon which mass emerges.

In fact, in physics and philosophy of physics, the relation of emergence is generally regarded as a generic relation which still has to be further interpreted, and is even consistent with reductionism (see e.g. [4, 7, 19]). Following this distinction, the extant literature has discussions on two different types of emergence, that have been called 'ontological' and 'epistemic' emergence.

Ontological emergence—the 'philosopher's definition'—is metaphysical in nature: the question being whether there are complex entities, properties, processes, etc, in the world, whose existence is dependent on but distinct from the existence of lower level entities, properties, etc. As an example, one could ask about the existence of minds over and above the existence of brains, in a dualistic sense of mind as a different, non-material kind of substance. But even without going so far as to endorse substance dualism, one could endorse the ontological emergence of properties. This would be the case of Chalmers' famous view on consciousness as a genuinely emergent property [5].

Epistemic emergence, on the other hand, stands for the (different) claim that properties, entities, or descriptions of higher level systems are unpredictable from those of the underlying microscopic subsystems, because the former cannot be completely expressed by the theories describing the lower levels. Thus, for many authors, although in principle complex molecules are 'made of' atoms described by quantum mechanics, the macroscopic concept of molecule would purportedly not be translatable to properties that can be seen at the level of the atomic description (e.g.

[27]). This kind of emergence doesn't deal with the ultimate ontology of systems or parts, but instead with how these systems can or cannot be described and modelled.

In this work, for the case of mass, we are interested in ontological emergence, but here too there are distinctions to be made. Whereas Chalmers proposes a sort of substance dualism, what we can call the strong emergence of mind from matter, other authors have proposed what has been called weak emergence. There are several notions of weak ontological emergence in the literature (the interested reader should consult [36]), but generally speaking they are all characterised for being compatible with some form of epistemic reduction. These notions of weak emergence naturally lead to property dualism. To wit: while minds are 'made of' brains and their neurons, a mental property or state (such as being in pain) does not reduce ontologically to a physical property or state (such as a given configuration of neurons firing), even if the physical state can explain its presence.

The clearest way in which this idea has been cashed out is by means of an *analogy* with the determinable/determinate distinction, or more precisely, by emphasising the distinction between a property and its instances. Starting with Macdonald and Macdonald [21], several authors, such as Yablo [39], Macdonald and Macdonald [22], and more recently Macdonald and Macdonald [23], Macdonald [24], Wilson [36], have used this approach to tackle the issue of the emergence of the mental, and of mental causation. In an analogous way to how the property 'being red' can be instantiated by many different shades of red, such as scarlet, burgundy, etc., every instantiation of a mental property can be considered to be identical to a certain physical property, while at the same time belonging, qua property, to an ontologically different class than any given physical property.

In order for this framework to succeed, the emergent properties have to fulfill a certain number of conditions. First, it is normally agreed upon that emergent properties should belong to a higher organizational level than the properties from which they emerge. This is clearly the case for the purported emergence of minds from brains. A second condition is what is known as supervenience: a type of dependence of higher level properties on lower level ones such that changes at the upper level necessitate changes at the lower level. The third important condition is multiple realizability: given a state or property at the higher level, there are many possible lower level states or properties that could instantiate the higher level property. Thus, the change of a mental state from e.g. surprise to pain would necessitate a change in the lower level neuronal configuration, and each mental state would be instantiated in many possible ways at the lower level, just as the red property could be instantiated by a diversity of shades of this color.

Notice that this proposal dissolves any issue with downward causation (see e.g. [15]). Indeed, this dissolution is the main motivation for [22]. The distinction between a property and its instantiation makes it possible that the higher level property can play a causal role *in virtue of being identical* to a given instantiation at the lower level. Thus, the instantiation of a mental state *is* a physical state, and the causal closure of the physical world does not enter into conflict with the emergence of mental states [24], while at the same time avoiding the trap of epiphenomenalism. This is relevant for us, as it allows us to avoid entering into the fraught metaphysics of causation.

Needless to say, this is only one out of many definition of ‘emergence’ in the philosophical literature. Here we are not interested in the differences or on the relative merits of each other with respect to absolute criteria. Macdonald and Macdonald [21] concept is especially useful for us to describe mass, as (i) it is a notion of emergence that is weak in the sense of not being a substance dualism, (ii) related to this, it is a framework for the emergence of properties from lower level properties, and (iii) it is nonetheless a proposal of ontological as opposed to epistemic emergence: the very theory of SR we analyze describes the properties of mass and energy and their relationship, but—we claim—the nature of these two properties is so different that it warrants their separation in two mutually exclusive ontological classes.

From now on, we make use of this concept of weak emergence, taking it from the realm of philosophy of mind to that of fundamental physics.

4 Mass as an Emergent Property

In order to motivate our interpretation, let us start with the example of the relativistic ideal gas, which on one hand illustrates the relationship between mass and energy, and on the other shows that the invariant mass of the compound exceeds the sum of the invariant mass of its components[4].

Let us consider a confined ideal gas, composed of non-interacting molecules, each with its own mass and energy. A trivial application of Eq. (1) ensures that

$$\sqrt{(\sum_i E_i/c)^2 - (\sum_i p_i)^2} = Mc, \tag{2}$$

where M is the invariant mass of the whole system, and the sum runs over all its components.

This result must hold for every reference frame, particularly for the one in which the gas as a whole is at rest; which implies that $\sum_i p_i = 0$ and the previous equation reduces to $\sum_i E_i = Mc^2$. Therefore the total mass of the gas corresponds to:

$$M = \frac{1}{c^2}[E_1 + E_2 + E_3 + ...] = \frac{1}{c^2}[m_1\gamma_1 + m_2\gamma_2 + m_3\gamma_3 + ...] \tag{3}$$

where m_i corresponds to the rest-mass of the ith component’s body. We can expand the previous sum in the limit of low velocities, i.e $(\frac{v_i}{c})^2 << 1$. In this case the previous result can be written in the following way:

$$M \approx [m_1 + m_2 + m_3 + ...] + \frac{1}{c^2}[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + ...] \tag{4}$$

[4] Already in 1913 Alexander Bogdánov, the unrecognized father of systems theory,while studying the widely used emergentist aphorism ‘the whole is greater than the sum of its parts’, had noted that according to contemporary physics “...the weight of a sack of potatoes must not be exactly equal to the sum of the separately measured weights of each spud and the sack; their mutual attraction alters, depending upon their spatial situation, this sum.”

or, for convenience

$$M \approx M_0 + M_k \tag{5}$$

where $M_0 = m_1 + m_2 + m_3 + \cdots$ is the mass term associated with the sum of the components' invariant masses, and $M_k = \frac{1}{c^2}[\frac{1}{2}m_1 v_1^2 + \frac{1}{2}(1/2)m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \cdots]$ is the mass term associated with the components' kinetic energy in a particular frame of reference (the one that leaves the total gas at rest), at the limit of low energies.

Notice that the kinetic terms are 'relational terms', in the sense that their numerical values are relative to the chosen reference frame, although their combined sum is invariant under that choice. Thus the intrinsic invariant mass of a compound depends on both the intrinsic invariant rest masses of its components, and on an invariant aggregation of the relational (individually not invariant) energies of these components.

With small modifications, a similar argument would also work for a gas composed by *massless* particles, such as photons. Thus, even though each photon is massless, the energy of each photon is frame dependent (as its frequency changes with the reference frame) in analogy to the energy of each molecule in the relativistic gas and, as in the case of a gas composed by massive particles, a gas composed of photons in a box has an associated mass, made out of the sum of the kinetic energy of the components which as a whole is invariant under Lorentz transformations (i.e. is frame independent).

Already at this point it is possible to see a gist of emergence, as M not only (i) is literally 'more than the sum of (the rest mass of) its parts'; but also (ii) is real (Lorentz invariant), (iii) supervenes on relational properties of parts, and (iv) is multiple realized. Additionally, energy corresponds to a property of the system described at a basal level (the parts of the system and their relations), while mass corresponds to the description of the system at a higher level (the system as a whole). Let us delve further into this.

In his proposal against the reality (or fundamentality) of energy, Lange has to deal with some very well known examples of mass-energy conversion, such as the ideal gas of particles. For this, he proposes a useful distinction, based on what can be called the level of analysis. Thus, to analyse the mass and energy of a system of particles, the description can be made at the system level, or at the component level. At the component level each particle has a certain mass and kinetic energy, whereas at the system level part of this kinetic energy contributes to the whole mass of the system. This distinction does not suffice to avoid the issues of Lange's interpretation when dealing e.g. with a totally inelastic collision [6], but they do provide an insight that is useful for the emergentist interpretation.

In order to present our proposal we begin by using Lange's distinction in order to separate two a priori different cases. Mass as a property can be assigned to (i) objects that are composed so that they can be considered systems, or (ii) fundamental objects that cannot be so decomposed. The distinction is important for our purposes because the only way to obtain a frame invariant value of energy that can contribute to mass is to consider systems and not isolated objects. In other words, as in the example of the ideal gas, it is only when we deal with whole systems that energy can make mass increase.

Consider case (i), that of the mass of a composed object. Here it is straightforward to make the case for mass as an emergent property. We propose that mass, as a determinable property of a system, its determined in value by the energy of the underlying configuration. Using Lange's distinction, if we consider the mass of the whole system, there will be contributions from the mass of its components, and also their kinetic and potential energies, i.e. the energy of their interactions. Each of these terms can be zero—for a gas of photons there is no mass of the components, for an ideal gas there are no interactions—but the general principle is that the mass of the system is determined by the energy of the configuration.

Thus, the mass of a gas of photons totally supervenes on the energy of its components. Not only that, but a given value for the mass of the system is determined by one of a myriad possible photon configurations, so that mass as a property is multiply realized. Mass—that is to say, the inertia of a composed object—can in this way be seen as emerging from the energy of the components, including the energy of their interactions.

The analogy with the alleged emergence of mental properties is easy to see. Mass would be analogous to a mental property (e.g. pain, or happiness), whereas energy would be the analogous to a physical property (e.g. a certain neuronal state). A given instance of the mass of a system is identical to the energy of a given configuration of components, in the same way as a given instance of a mental property is identical with a physical property, instantiated by a certain neuronal configuration. Such as mental properties would be independent of but instantiated by physical properties; mass, as a determinable property, is independent (and in particular different) from energy, but each instance of mass has a value determined by energy.

Let us consider now case (ii) as defined above. As mentioned in Sect. 2, the conceivable existence of fundamental particles with nonzero rest mass can be in principle problematic for Bondi and Spurgin [3]. Notice that the emergence interpretation is related to this no-conversion interpretation. Mass and energy are considered ontologically different properties, and a process of emergence is not one of conversion. This being the case, it is natural that an issue with fundamental masses also appears for the emergence interpretation, although in this case for slightly different reasons.

Bondi and Spurgin [3] proposal has particular problems dealing with processes of total appearance or disappearance of mass, such as pair creation or annihilation, and the problem becomes more pressing when dealing with fundamental particles, as it is not easy to see a smooth way in which this fundamental quantity of mass or energy can be exchanged without conversion. In our case, however, we can still characterize this as a process of emergence (or disappearance) of mass from (or into) energy.

If mass emerges from energy, the changes in the configuration or structure of relations of the energies of a system would superveniently result in a change in the mass of the system. By analogy, if the development of some configurations of neurons lies underneath the emergence of consciousness in an organism, a strong alteration of this configuration (e.g., disconnecting neurons) would make consciousness de-emerge (disappear). If we assume that mass and energy correspond to properties of two different levels of organization of systems, the appearance and disappearance of mass is more naturally understood as the consequence, at a higher level of organization (the level of the whole system), of a change in the configuration of energy at a lower level of organization (the level of the parts). This view yields thus a natural interpretation of the pair annihilation processes in particle physics.

The challenge remains, however, of providing an adequate account of the existence of fundamental particles with nonzero rest mass within our emergentist interpretation. Indeed, if fundamental particles have mass, we would have trouble speaking of this fundamental mass as emergent. There are two answers to this challenge.

First, our best available theory of matter, the standard model of particle physics, shows that mass is not an intrinsic property of fundamental particles, but that it is instead dynamically acquired through interactions with the Higgs field (this is true also for the Higgs particle itself, as the Higgs is a self-interacting field[5]). This is in fact one of the motivations we have to describe mass as emergent in general, as the Higgs mechanism presents a clear case of mass not being a fundamental property, but a consequence of interactions—a relational property which can be interpreted as emerging from the energy related to an interaction. We delve into these facts in the next section.

However, the fact that in our world there are no fundamental masses does not change the fact that this would be a possibility: SR is a general framework that applies to a wide range of possible phenomena besides what our particle physics says, and in particular, as a framework, it does not assume anything about the nature of matter (for this, see [11, 12, 29]). Because of this, as a second line of defense we claim that, once any fundamental mass is taken into account, every other mass in the world emerges from the energy of the fundamental objects and their interactions. In other words, once the inertia of the basic building blocks is accounted for, all the remaining inertia in the world emerges from energy.

A usual step directed at characterizing emergence is the distinction between 'emergent' and 'resultant' [15], or 'additive' and 'aggregative' [37] properties. These latter are collective properties that are 'not more than' the conjunction of the properties of parts. That is, a resultant property is not something ontologically new in the world, but merely the aggregate of the properties of parts. Resultant properties can be ontologically reduced to the properties of parts, while emergent properties cannot. However, we gain little by forbidding 'collections of aggregates' from our list of candidates for emergent properties. Instead, it turns out to be more fruitful to focus on the concept of 'novelty'. The whole concept of emergence requires the higher level property to be 'novel' (or different) from the lower level *in some sense*, otherwise the idea of emergence is pointless. As already emphasised by Wimsatt

5 Because there has been some confusion about this, technically, for a standard quartic Higgs potential, the Higgs mass term in the standard model Lagrangian is proportional to the vev at tree level. This mass term, which reads $m^2\phi^\dagger\phi = \frac{\lambda}{6}v^2\phi^\dagger\phi$ (λ being the Higgs self-interaction coupling, ϕ the Higgs field, and v the vev) reads the same way as an interaction term between the Higgs field and a constant external scalar field. Of course, as with all equalities, this equality can be (and has been) interpreted the other way around: as implying that the mass of the Higgs is the originator of the vev of the field—but our reading of it is just as valid, and consistent with the rest of the standard model.

[37, 38]; Mitchell [25], certain kinds of aggregates could give rise to novel properties. However, is not completely clear *in what sense* the higher level property has to be different from the lower level properties.

A more fine-grained distinction than the one provided by aggregativity is the one introduced by Armstrong, who famously imposed that an emergent property has to be anomoeomerous.

> A property is homoeomerous if, and only if, for all particulars x which have that property, then for all parts y of x, y also has that property. If a property is not homoeomerous, then it is anomoeomerous. [1, p. 68f].

Anomoeomericity would amount to *evidence* of emergence, but it is not a necessary *requirement* for emergence. Consider mass as arising from a certain configuration of energy. In this case, something new in the world appears, a new property different from energy. Mass terms genuinely appear from the kinetic energy of the constituents in a relativistic gas: these terms are different to the sum of rest masses of the components, and they contribute to the inertia of the whole gas. That is, the rest mass of the whole is constituted by the individual rest masses together with a contribution that, at low velocities, can be clearly identified with the kinetic energy of the component particles. The same property that is present in each component nonetheless emerges in the whole; this is a case of an ontological emergent property which is not qualitatively, but instead quantitatively, novel. Inspired by Armstrong, such a feature could be called *homoeomerous* emergence.

Furthermore, according to SR, a collection of particles, and systems in general, have a mass value associated to their total energy. This is particularly important in the case of bound states, where particles are bound together by some potential energy (gravitational, electrical, nuclear). In these cases, the total mass of the bound state is made from the individual masses of its parts, their kinetic energies, and the potential energies binding them. Within our current scientific picture on the nature of matter, potential energy is responsible for 99% of the total mass of protons and neutrons, where the binding mechanism is driven by strong nuclear forces between the quarks that compose them. That is to say, the sum of the masses of the component quarks is only about a hundredth of total mass of protons and neutrons. Given that most matter is made of atoms, and that in turn most of the atomic mass stems from the mass of the nucleons, it is fair to say that most of the mass we observe emerges from this binding energy. If we consider separately the three quarks composing the proton versus the proton itself, it is clear that some new property, namely the total mass, appeared out of the interaction of the quarks.

Thus, the requirement of ‘anomoeomericity’ plays the role of ensuring novelty. However, novelty can be qualitative *or*, as in the case of SR, only quantitative—inertia increasing through interactions is still a case of novelty, but not necessarily a new property in the world. Nevertheless, with this we are not implying that all possible emergent properties must be instantiated all the way down to their components. In other words –using again the analogy with the mind– we are not claiming, for instance, that all molecules composing a conscious brain are necessarily also conscious. Furthermore, we are not implying that all cases of emergence reduce to quantitative emergence.

To the best of our knowledge, the homoeomerous approach to emergence has been absent in the specialized literature on the topic until the arrival of mass as an example of an ontological emergent property.

5 Emergence of Mass in the Standard Model

As mentioned in the previous section, the Higgs mechanism within the standard model plays a double role in our approach: it serves as an inspiration, often used by physicists as an example of emergence (a notable example is the aforementioned [35])[6]. Additionally, it provides us with a working model (and a realistic one) of how all the mass in the universe can be considered to emerge from energy—that is to say, the consistency of a universe without any fundamental masses. So, even if the standard model is only a small subset of all the physics involved with $E = mc^2$, it is worth to take a detour to use it as an example which lends strength to our proposal.

Historically, the success of quantum mechanics and of the relativistic quantum theory in terms of fields made clear that something unsettling lies behind the idea of mass as an intrinsic property of particles (considered in this context as quantum field excitations). The mass terms in the Lagrangian are not invariant under the (gauge) symmetries of the standard model, which means that arbitrary choices in the mathematics would have measurable consequences: an unacceptable result. The so-called Higgs mechanism is the most simple solution to this problem, but such a solution comes with the cost of renouncing to the idea that the mass of an object is an intrinsic and fundamental property.

Roughly speaking, a gauge symmetry is a mathematical freedom at the level of the description of a field theory that leaves the physical observables invariant. These symmetries have physical consequences, and one of them is that the force carriers, the so-called gauge bosons, must have zero mass. Because this is not borne out experimentally, it becomes necessary to devise a mass-generating mechanism—the Higgs mechanism, which preserves the gauge symmetry and dynamically generates not only the mass of the force carriers, but all the masses in the standard model. This mechanism, which has recently received strong empirical support, provides us with further tools to understand the nature of mass and to test our ideas.

The Higgs mechanism represents, by all accounts, an incontrovertible case for the genuine ontological emergence of mass. In the standard model, all particles initially have no mass, but gain it due to interaction with a scalar field (the Higgs field), which, being a scalar spinless field, is allowed to have a vacuum expectation value (vev). The mass of every particle in the SM is generated by the interaction of each field with this vev, i.e. with the remnant energy associated with the ground-state of a scalar field (its vacuum), leading to a mass-like term in the effective Lagrangian

[6] Of course, physicists' use of the word 'emergence' is more loose than that of philosophers, but their views are naturally important for the interpretation of physics, and provide an intuition pump for us.

of the particles. Notice that this is also true when considering the Higgs field itself, which is self-interacting.

The value of the Higgs vacuum expectation value is a *contingent* quantity as far as our present knowledge of physics indicates. It is easy to envision a world where this vev is zero, as that was effectively the case in our universe shortly after the big bang, when the mean energy was greater than the electroweak Spontaneous Symmetry Breaking (SSB) scale. In these circumstances, all the laws of physics would be exactly the same as those that we know today, but such a world is very different than our present world, as the electroweak SSB leads to the breaking of the so-called *conformal* symmetry [32]. Shortly after the big bang, there were no masses. All matter fields, and all interaction (gauge) fields were massless, such as is the case for the photon in our present world. Thus, mass is not a necessary property in our world, whereas energy is. Most relevantly, the properties of fields having mass are radically different from those of massless fields, precisely because of this additional conformal symmetry, which is broken by massive fields. All massless fields propagate at the speed of light, along a light cone. It is only with the appearence of mass that fields that propagate slower than the speed of light start furnishing the world.

The Higgs mechanism shows that mass and energy are not the same property in the standard model. A possible reading of the situation would be that mass, generated from energy by this mechanism, is nothing over and above energy—what would constitute the reduction of the concept of mass to that of energy, at least in this context. However, mass is not an illusion or a convenient way to represent energy. The radically different behaviour of massive as opposed to zero mass quantum fields and particles, i.e. different causal behaviour, and also different number of degrees of freedom [32], allows us to consider mass as a very distinct, causally powerful property that exist in our world.

Now, it is important to distinguish two different notions of emergence when we speak of emergence of mass via the Higgs mechanism. First, there is *synchronic* emergence [8, 14, 30]: at any point in time, the mass of every particle in the universe is generated via interactions with the Higgs vev, in such a way that every example of mass we observe is due to an interaction, and is instantiated by a certain interaction energy. This is the notion of emergence explored in this work. Second, the emergence of mass *in time*, from an initial condition just after the big bang where no masses were present, as discussed in the paragraph above. This has been called *diachronic* emergence, and is not the focus here. What is important about the dyachronic emergence of mass in the universe is that it shows in a clear way the ontological independence of mass and energy in our world.

As a further example of the compatibility of SM with the notion of emergence of mass and the role played by SSB, an example aptly emphasized by Wilczek [35], we can consider chiral symmetry breaking in the theory of the strong nuclear force. This is another case of SSB, and, even though the broken symmetry in this case is only an approximate one, it is responsible for most of the visible mass in the universe. The masses of the three most common quarks u, d, and s are low enough to make the approximation of massless fields a useful and successful one. Without symmetry breaking, the mass of mesons and baryons constituted by these quarks should also be of order zero (or more precisely, of the order of the small quark masses). As this symmetry is spontaneously broken, one observes instead a family of light mesons, and further families of very massive mesons and baryons. In fact, this chiral symmetry breaking is responsible for 99% of the mass of protons and neutrons, as discussed in the previous section. In the simplest picture, this extra mass is seen as the result of the energy binding the quarks together in a nucleon. Thus we have a clear emergence of most of the universe's mass from the binding energy provided by the strong nuclear force.

Mass and energy being different, one could ask if there can be conversion of energy into mass or vice-versa. In the case of the standard model, looking e.g. to the chiral SSB, it is clear that energy does not convert into mass, but that the mass of an object (e.g. a composed object such as a proton) depends upon its energy-content. Conversely, in an event such as pair annihilation, it is not the case that the mass of the pair of particles converts into energy; what happens instead is an interaction process having different initial and final total mass, while having the same amount of initial and final energy.

A conversion is a physical process that takes place among two magnitudes of the same ontological level—in our case, if mass could be converted into energy and vice-versa, then both properties would belong to the same level of reality, and there would be a process ruled by laws of physics regulating the passage from one to the other. Synchronous emergence is something different, it is not a physical process but a relation among two different magnitudes that belong to different ontological levels, as one ontologically depends on the other in the strict sense that the dependent property would not exist if the primordial property is absent. A classical example is that the brain does not convert into a mind nor the other way around, but the mind *emerges* from the brain.

We have all the elements to establish that, within the SM, mass and energy fulfil all standard features of an emergent relation between properties, namely:

1 *We deal with two ontologically different properties.* As pointed in the previous paragraph, mass and energy are two different properties, in fact there were times in our own universe in which there was energy, fields, and particles, but none of them had mass. Notice that we did not have to appeal to new causal powers to argue for this.
2 *Such different properties are describable at different organizational levels.* It is clear from the examples of the Higgs mechanism and of chiral SSB that the appeareance of mass is due to the collective behaviour of quantum fields. That is to say, mass is a property that appears at the level of description of wholes and not of parts. The mass that is generated through the Higgs mechanism is due to the interactions between the Higgs vev and the diverse quantum fields that make up the world; the mass of the proton is made up (mainly) of the interaction energy of its quark components. Similar considerations apply to the mass of bound states in general.
3 *One property supervenes on the other.* This is again clear in the case of the Higgs mechanism. Mass supervenes on the energy of the interaction between the ground-state of the Higgs field and the diverse quantum fields that make up the

world—a change in the value of mass implies a change in the interaction strength between the Higgs vev and the field under consideration.

4 *Multiple realizability* The Higgs mechanism depends on the coupling of diverse fields to the Higgs fields, and once this coupling is set the value of the mass of the field is fixed—there is no multiple realizability in this sense. However, in a broader sense, for a particle to have a given mass does not imply a full identification of the underlying physical situation, as particle mass depends on many parameters: the Higgs vev, the coupling with the Higgs, the electroweak charge of the field, and so on. Multiple realizability is much more clear in the case of the chiral SSB, where inside a proton myriad of changes happen continously, due to the strong interaction: the identity of quarks inside a proton is not fixed, for example [32]. A proton is realized, at each moment, by a different subjacent configuration of up quarks, down quarks, and gluons.

Within the standard model of particle physics, then, mass is best understood as an emergent and relational property of objects rather that a fundamental and intrinsic property of these objects, and emerges upon energy (e.g. the energy term in the Lagrangian due to the interaction between the Higgs vev and all fields, or the interaction energy between quarks inside a nucleon).

6 Conclusions

After more than a century since the equation $E = mc^2$ was established, there is still not agreement on its exact physical meaning. The most straightforward interpretation, that which equates or reduces mass to energy, is unattainable, as has been extensively argued by several authors. This has led to the development of a diversity of interpretations, each having its own set of issues. In this work, we propose a new interpretation, based on the concept of ontological emergence of properties.

According to our interpretation, mass is associated to the emergence of localizable inertial properties within the framework of special relativity. We are not the first to think along these lines, as the idea of mass as a non-fundamental, emergent property has been entertained by various authors in the past, both in physics and in philosophy. But we are the first to connect all the dots, in the sense of at the same time (i) showing in what sense mass can be considered emergent from energy, (ii) showing how this idea is valid both for special relativity, and to the more specific theory of the standard model of particle physics, and (iii) providing a consistent interpretation of $E = mc^2$.

In this work we show how the idea of mass as an emergent property is eminently compatible with well-known proposals on the emergence of properties, which were originally introduced to deal with the emergence of mental states and properties, but which demonstrate its versatility by describing emergence in a new context—a more fundamental one.

We have also explored how this notion of emergence is not only relevant for special relativity as a general framework, but also allows us to interpret the role of mass in the standard model of particle physics, through the famed Higgs mechanism.

Although it is not per se surprising that the same approach would work in both cases—after all the standard model *is* a relativistic theory—this general consistency increases the strength of the emergence interpretation.

Finally, we think this interpretation of $E = mc^2$ solves all the issues with previous proposals. It shows how energy and mass, while being ontologically distinct properties, are intimately related. In particular, it shows how mass supervenes and depends on energy, while not being directly convertible into energy as it was proposed in previous interpretations.

This work not only closes a decades-long debate, but it also opens the door for the analysis of emergentist ideas from our most general and fundamental theoretical frameworks. We think this might in the end prove to be its most fruitful contribution.

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